Section 8.1

The first step to computing an integral is to know the basic indefinite integral formulas:

Basic Integral Formulas:

$$\int k\,dx = kx + C \qquad \int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax + C \qquad \int \cos ax\,dx = \frac{1}{a}\sin ax + C$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax + C \qquad \int \csc^2 ax\,dx = -\frac{1}{a}\cot ax + C$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax + C \qquad \int \csc ax\cot ax\,dx = -\frac{1}{a}\csc ax + C$$

$$\int \frac{1}{x}dx = \ln|x| + C \qquad \int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \arcsin\frac{x}{a} + C \qquad \int \frac{1}{a^2+x^2}dx = \frac{1}{a}\arctan\frac{x}{a} + C$$

$$\int \frac{1}{x\sqrt{x^2-a^2}}dx = \frac{1}{a}\text{arcsec}\left|\frac{x}{a}\right| + C$$

The "Move Stuff Around" Approach.

Our approach to integrals in this section is to rewrite the integrand, typically using algebra or by multiplying the integrand by 1, in such a way as to relate back to the integrals found in our basic integral table. Our first applications will be to add some missing integrands to the above table:

Example 8.1.1. Compute $\int \tan x\,dx$.

Example 8.1.2. Compute $\int \sec x \, dx$.

$$\int \tan ax \, dx = \frac{1}{a} \ln \left| \sec ax \right| + C \qquad \int \sec ax \, dx = \frac{1}{a} \ln \left| \sec ax + \tan ax \right| + C$$

$$\int \cot ax \, dx = \frac{1}{a} \ln \left| \sin ax \right| + C \qquad \int \csc ax \, dx = -\frac{1}{a} \ln \left| \csc ax + \cot ax \right| + C$$

Let's look at some more rewriting of integrands.

Example 8.1.3. Compute $\displaystyle\int \frac{dx}{e^x + e^{-x}}$.

Example 8.1.4. Compute $\displaystyle\int \frac{\cos x + \sin^3 x}{\sec x} dx$.

Example 8.1.5. Compute $\int_2^4 \frac{x^2+2}{x-1}dx$.

Example 8.1.6. Compute $\int \frac{x-2}{x^2+6x+13}dx$.